



# Curriculum for the Master's Program in Mathematics

Aalborg University  
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**Preface:**

Pursuant to Act 261 of March 18, 2015 on Universities (the University Act) with subsequent changes, the following curriculum for the Master's program in Mathematics is stipulated. The program also follows the Joint program regulations and the Examination Policies and Procedures for the Faculty of IT and Design, The Faculty of Engineering and Science, and The Faculty of Medicine.

**Table of Contents**

Chapter 1: Legal Basis of the Curriculum, etc. ....	2
1.1 Basis in ministerial orders .....	2
1.2 Faculty affiliation .....	2
1.3 Board of Studies affiliation.....	2
1.4 Body of External Examiners.....	2
Chapter 2: Admission, Degree Designation, Program Duration.....	2
and Competence Profile.....	2
2.1 Admission .....	2
2.2 Degree designation in Danish and English.....	2
2.3 The program's specification in ECTS credits.....	2
2.4 Competence profile on the diploma.....	3
2.5 Competence profile of the program: .....	3
Chapter 3: Content and Organization of the Program .....	4
3.0 Course in Problem Based Learning and Student Responsibilities at Aalborg University.....	9
3.1 Descriptions of modules.....	10
3.1.1 1'st semester, MAT7A. Applied Mathematics .....	10
3.1.2 2'nd semester, MAT8A. Applied Mathematics .....	20
3.1.3 3'rd semester MAT9A, Applied Mathematics .....	32
3.1.4 4'th semester MAT10A, Applied Mathematics .....	36
3.1.5 MAT7M Mathematics as a central subject (Matematik som centralt fag).....	37
3.1.6 MAT8M Mathematics as a central subject (Matematik som centralt fag).....	41
3.1.7 MAT9M Mathematics as a central subject (Matematik som centralt fag).....	44
3.1.8 MAT10M Mathematics as a central subject (Matematik som centralt fag).....	44
3.1.9 Mathematics as a central/elective subject (Matematik som sidefag) .....	44
Chapter 4: Entry into Force, Interim Provisions and Revision.....	44
Chapter 5: Other Provisions.....	44
5.1 Rules concerning written work, including the Master's Thesis .....	44
5.2 Rules concerning credit transfer (merit), including the possibility for choice of modules that are part of another program at a university in Denmark or abroad .....	45
5.3 Rules for examinations.....	45
5.4 Exemption.....	45
5.5 Rules and requirements for the reading of texts .....	45
5.6 Additional information.....	45

## ***Chapter 1: Legal Basis of the Curriculum, etc.***

### **1.1 Basis in ministerial orders**

The Master's program in Mathematics is organised in accordance with the Ministry of Higher Education and Science's Ministerial Order no. 1328 of November 15, 2016 on Bachelor's and Master's Programs at Universities (the Ministerial Order of the Study Programs) and Ministerial Order no. 1062 of June 30, 2016 on University Examinations (the Examination Order) with subsequent changes. Further reference is made to Ministerial Order no. 111 of January 30, 2017 (the Admission Order) and Ministerial Order no. 114 of February 3, 2015 (the Grading Scale Order) with subsequent changes.

### **1.2 Faculty affiliation**

The Master's program falls under the Faculty of Engineering and Science, Aalborg University.

### **1.3 Board of Studies affiliation**

The Master's program falls under the Board of Studies for Mathematics, Physics and Nanotechnology.

### **1.4 Body of External Examiners**

The Master's program is associated with the Body of External Examiners for Mathematics (Censorkorpset for matematik).

## ***Chapter 2: Admission, Degree Designation, Program Duration and Competence Profile***

### **2.1 Admission**

#### **Applicants with a legal claim to admission (retskrav):**

Applicants with one of the following degrees are entitled to admission:

- Bachelor (BSc) of Science in Mathematics, Aalborg University

#### **Applicants without legal claim to admission:**

Students with another Bachelor degree may, upon application to the Board of Studies, be admitted following a specific academic assessment if the applicant is considered as having comparable educational prerequisites. The University can stipulate requirements concerning conducting additional exams prior to the start of study.

### **2.2 Degree designation in Danish and English**

The Master's program entitles the graduate to the designation:

Cand.scient.(candidatus/candidata scientiarum) i matematik. The English designation is: Master of Science (MSc) in Mathematics.

Or

Cand.scient. (candidatus/candidata scientiarum) i matematik og [sidefag]. The English designation is: Master of Science (MSc) in Mathematics and [Minor Subject].

### **2.3 The program's specification in ECTS credits**

The Master's program is a 2-year, research-based, full-time study program. The program is set to 120 ECTS credits.

## 2.4 Competence profile on the diploma

The following competence profile will appear on the diploma:

A Candidatus graduate has the following competency profile:

A Candidatus graduate has competencies that have been acquired via a course of study that has taken place in a research environment.

A Candidatus graduate is qualified for employment on the labour market on the basis of his or her academic discipline as well as for further research (PhD programs). A Candidatus graduate has, compared to a Bachelor, developed his or her academic knowledge and independence so as to be able to apply scientific theory and method on an independent basis within both an academic and a professional context.

## 2.5 Competence profile of the program:

### Students graduating as Masters of Mathematics

#### Knowledge

- are well-oriented in the foundations of key mathematical disciplines including mathematical analysis, algebra, geometry, probability and statistics

#### Skills

- are able to independently identify, formulate and analyse mathematical problems employing theory and methodology from the mathematical sciences
- are able to independently choose relevant methods and tools from various mathematical areas and to motivate this choice
- are able to disseminate scientific knowledge and to discuss applications of methods from the mathematical sciences

#### Competencies

- are able to ponder about central mathematical insights, methods and tools and to identify problems amenable to mathematical treatment
- are able to manage complex work and development scenarios that may require new strategies in order to make progress
- are able to independently take responsibility for professional development and specialization

### Moreover, graduates within Applied Mathematics

#### Knowledge

- have acquired a profound understanding within one or a few mathematical areas linking up to international research level

#### Skills

- can apply techniques of mathematical modelling to theories and problems originating in scientific areas from outside of Mathematics
- are able to choose relevant mathematical theories to problems that originate in, for example, engineering, computer science or economics, to develop them and to make use of them in the original applied context

Competencies

- are able to launch and to perform professional and responsible scientific collaboration with peers from within and from outside of Mathematics

**Moreover, students graduating in Mathematics in combination within a second subject**  
Knowledge

- have acquired a broad view into mathematical theories and methods within several mathematical areas and their mutual connections

Skills

- are able to disseminate scientific knowledge to non-experts and to reflect about best practices how to achieve good understanding

Competencies

- are able to identify requirements for their further development of scientific knowledge and methodology and to structure ways of achieving requested insights

***Chapter 3: Content and Organization of the Program***

The program is structured in modules and organized as a problem-based study. A module is a program element or a group of program elements, which aims to give students a set of professional skills within a fixed time frame specified in ECTS credits, and concluding with one or more examinations within specific exam periods. Examinations are defined in the curriculum.

The program is based on a combination of academic, problem-oriented and interdisciplinary approaches and organized based on the following work and evaluation methods that combine skills and reflection:

- lectures
- classroom instruction
- project work
- workshops
- exercises (individually and in groups)
- teacher feedback
- reflection
- portfolio work

**Overview of the programs:**

All modules are assessed through individual grading according to the 7-point scale *or* Pass/Fail. All modules are assessed by external examination (external grading) or internal examination (internal grading or by assessment by the supervisor only).

**General provisions concerning elective courses:**

- Only a limited number of elective courses from the list of courses will be offered at each semester.
- Students can only participate once in a course with a given title. In particular, they cannot follow a course if they have previously participated in a course with the same title as part of a bachelor program.

## Applied Mathematics

Semester	Module	ECTS	Assessment	Exam	
1st MAT7A	Project. Introductory Application Oriented Mathematics	15	7-point scale	External	
	<i>Elective courses (valgfag) – the students select courses equivalent to 15 ECTS:</i>				
	Introduction to Partial Differential Equations (MATØK5)	5	Pass/Fail	Internal	
	Numerical Analysis (MATØK7)	5	Pass/Fail	Internal	
	Manifolds – Differential Geometry and Topology	5	Pass/Fail	Internal	
	Optimization (MATØK5)	5	7-point scale	External	
	Measure Theory and Stochastic Processes (MATØK7)	5	Pass/Fail	Internal	
	Statistics for Duration Data	5	Pass/Fail	Internal	
	Topics in Statistical Science I	5	Pass/Fail	Internal	
	Topics in Statistical Science II	5	Pass/Fail	Internal	
	Information and Coding Theory (MATTEK7)	5	7-point scale	Internal	
	<i>Courses from different curricula, e.g. engineering or computer science, as described in the requirements for Applied Mathematics</i>				
	2nd MAT8A	Project. Intermediate Application Oriented Mathematics	15	7-point scale	Internal
		<i>Elective courses (valgfag) – the students select courses equivalent to 15 ECTS:</i>			
Bayesian Inference and Mixed Models (MATTEK8)		5	Pass/Fail	Internal	
Time Series and Econometrics (MATØK6)		5	7-point scale	Internal	
Spatial Statistics and Markov Chain Monte Carlo Methods (MATTEK6)		5	Pass/Fail	Internal	
Graph Theory		5	Pass/Fail	Internal	
Coding Theory		5	Pass/Fail	Internal	
Applied Harmonic Analysis (MATTEK4)			Pass/Fail	Internal	
Operators on Hilbert Spaces		5	Pass/Fail	Internal	
Algebraic Topology		5	Pass/Fail	Internal	
Quantitative Finance and Computational Statistics (MATØK8)		5	Pass/Fail	Internal	
Financial Engineering (MATØK6)		5	Pass/Fail	Internal	
Data Mining (MATØK8)		5	Pass/Fail	Internal	
<i>Courses from different curricula, e.g. engineering or computer science, as described in the requirements for Applied Mathematics</i>					
3rd MAT9A  1	Project. Advanced Application Oriented Mathematics	20	7-point scale	Internal	
	<i>Elective courses (valgfag) – the students select courses equivalent to 10 ECTS. 10</i>				
	Introduction to Partial Differential Equations (MATØK5)	5	Pass/Fail	Internal	
	Numerical Analysis (MATØK7)	5	Pass/Fail	Internal	
	Manifolds – Differential Geometry and Topology	5	Pass/Fail	Internal	
	Optimization (MATØK5)	5	7-point scale	External	

	Measure Theory and Stochastic Processes (MATØK7)	5	Pass/Fail	Internal
	Statistics for Duration Data	5	Pass/Fail	Internal
	Topics in Statistical Science I	5	Pass/Fail	Internal
	Topics in Statistical Science II	5	Pass/Fail	Internal
	Topics in Algebraic Geometry and Commutative Algebra	5	Pass/Fail	Internal
	Topics in Applied Mathematical Analysis and Geometry	5	Pass/Fail	Internal
	<i>Courses from different curricula, e.g. engineering or computer science, as described in the requirements for Applied Mathematics.</i>			
3rd MAT9A or 2	Study at another university, must be approved by the Study Board	30	Transfer of credits	Transfer of credits
3rd MAT9A or 3	Long Master's Thesis <sup>1</sup> , must be approved by the Study Board	+30	7-point scale	External
4th MAT10A	Master's Thesis	30	7-point scale	External
Total		120		

The study board can cancel modules if the number of enrolled students is low.

A compulsory course in Problem Based Learning (PBL) and student responsibility is offered as an integrated part of all project modules to students not acquainted with PBL at Aalborg University.

<sup>1</sup> See module description for the Master's thesis. The long Master's thesis, which must be of experimental character, is prepared in the 3<sup>rd</sup> and 4<sup>th</sup> semesters; the extent is 60 ECTS.

**Mathematics as central subject in combination with a minor subject****Tofagsuddannelse med matematik som centralt fag, gymnasielærer.**

Example. NAT minor subject (for example Biology as a minor subject). The student has followed 60 ECTS on the minor subject at the BA-level.

Individual plans must be approved by the two Study Boards involved.

Semester	Module	ECTS	Assessment	Exam
1st MAT7M	Project. Statistical Modelling and Analysis	15	7-point scale	External
	Statistical Inference for Linear Models (MAT5)	5	Pass/fail	Internal
	Computer Algebra (MAT5)	5	Pass/fail	Internal
	Differential Geometry (MAT5)	5	7-point scale	Internal
2nd MAT8M	Project. Mathematics with Applications	15	7-point scale	Internal
	Integration Theory	5	7-point scale	External
	<i>Elective courses – the students select courses equivalent to 10 ECTS</i>			
	Bayesian Inference and Mixed Models (MATTEK8)	5	Pass/fail	Internal
	Time Series and Econometrics (MATØK6)	5	7-point scale	Internal
	Spatial Statistics and Markov Chain Monte Carlo Methods (MATTEK6)	5	Pass/fail	Internal
	Graph Theory	5	Pass/fail	Internal
	Coding Theory	5	Pass/fail	Internal
	Applied Harmonic Analysis (MATTEK4)		Pass/Fail	Internal
	Operators on Hilbert Spaces	5	Pass/fail	Internal
	Algebraic Topology	5	Pass/fail	Internal
	Quantitative Finance and Computational Statistics	5	Pass/fail	Internal
	Financial Engineering (MATØK6)	5	Pass/fail	Internal
	Data Mining (MATØK8)	5	Pass/fail	Internal
	3rd semester	Minor subject	30	
4th MAT10M	Master's Thesis in Mathematics	30	7-point scale	External
Total		120		

**Mathematics as minor subject in combination with central subject****Tofagsuddannelse med matematik som sidefag, gymnasielærer.**

Example. NAT central subject (for example Biology as a central subject). The student has followed 60 ECTS mathematics on the BA-level.

Individual plans must be approved by the two Study Boards involved.

Semester	Module	ECTS	Assessment	Exam
1st	Central subject (central fag)	30		
2nd	Central subject (central fag)	30		
3rd MAT7m	Project, Statistical Modelling and Analysis	15	7-point scale	External
	Statistical Inference for Linear Models (MAT5)	5	Pass/fail	Internal
	Computer Algebra (MAT5)	5	Pass/fail	Internal
	Differential Geometry (MAT5)	5	7-point scale	Internal
4th	Master's Thesis within the Central subject (centrale fag)	30	7-point scale	External
Total		120		

Example. HUM/SAMF/IDRÆT central subject. The student has followed 45 ECTS mathematics on the BA-level.

Individual plans must be approved by the two Study Boards involved.

Semester	Module	ECTS	Assessment	Exam
1st MAT3	Project, Ordinary Differential Equations	15	7-point scale	Internal
	Analyses I	5	7-point scale	External
	Linear Algebra with Applications	5	7-point scale	Internal
	Algebra 1: Groups	5	7-point scale	Internal
2nd MAT4	Project, Symmetry	10	7-point scale	External
	Probability Theory	5	7-point scale	Internal
	Analysis 2	5	7-point scale	Internal
	Algebra 2: Rings and Fields	5	7-point scale	External
	Complex Functions or The Didactics of Mathematics	5	Pass/fail	Internal
3rd MAT7m and central subject	Statistical Inference for Linear Models (MAT5)	5	Pass/fail	Internal
	Computer Algebra (MAT5)	5	Pass/fail	Internal
	Differential Geometry (MAT5)	5	7-point scale	Internal
	Central subject (central fag)	15		
4th	Central subject (central fag)	30		
5th	Master's Thesis within the Central subject (centrale fag)	30	7-point scale	External
Total		120		

### 3.0 Course in Problem Based Learning and Student Responsibilities at Aalborg University

<p><b>Title:</b> Problem Based Learning and Project Management (Problembaseret læring og projektledelse)</p>
<p><b>Objectives:</b> The objective is to make newly started Master students coming from institutions other than AAU prepared to enter the problem based learning environment at AAU and manage study projects in close collaboration with peers.</p>
<p><b>Type of instruction:</b> Three half day workshops centered around the individual student working with an individual challenge or curiosity in relation to using a PBL approach. Peer learning is also a hallmark, since the students will discuss and reflect their individual challenges/curiosities in a peer learning group.</p>
<p><b>Learning outcomes:</b> After completion of the course the student should be able to</p> <p><b>Day 1:</b></p> <ul style="list-style-type: none"> <li>- describe and discuss the Aalborg PBL model based on the three key words: group work, project work, problem orientation</li> <li>- identify an initial individual challenge when using a PBL approach</li> </ul> <p><b>Day2:</b></p> <ul style="list-style-type: none"> <li>- develop and practice peer feedback skills</li> <li>- practice collaborative learning in a group</li> <li>- design a plan of action to deal with an initial individual PBL challenge or curiosity</li> </ul> <p><b>Day 3:</b></p> <ul style="list-style-type: none"> <li>- practice presentation skills</li> <li>- practice critical skills when giving feedback to peers</li> <li>- reflect on own and peer skills in relation to PBL practice</li> </ul>
<p><b>Exam format:</b> Internal assessment during the course/class participation according to the rules in the Examination Policies and Procedures of Faculty of Engineering and Science, Aalborg University. In this case the assessment is primarily based on the oral performance during the course, which means that the student has to be active during the course time and participate in discussions. The course is an integrated part of the project for those not acquainted to the Aalborg PBL model, and is a precondition for participation in the project examination. In this way there will be no diploma for the course and it will not be visible on the academic transcripts.</p>
<p><b>Evaluation criteria:</b> The criteria for the evaluation are specified in the Joint program regulations.</p>

### 3.1 Descriptions of modules

#### 3.1.1 1'st semester, MAT7A. Applied Mathematics

##### 3.1.1.1 Project MAT7A

#### **Project: Introductory Application Oriented Mathematics/ Indledende anvendelsesorienteret matematik**

Objective: Students who have completed the module meet the following criteria within at least one central mathematical area:

Knowledge:

- are able to explain important introductory notions, results and theories connected to a central mathematical subject area
- can relate such results to application(s) outside the subject area

Skills:

- are able to apply a number of introductory methods and tools from a specific mathematical subject area
- can assess whether a specific result is valid and/or whether a specific method can be applied under prescribed circumstances
- are able to form a mathematical model describing a problem amenable to a mathematical investigation and to perform a mathematical analysis that sheds light on the initiating problem
- can under guidance select adequate mathematical methods and tools that help treating certain questions amenable to a mathematical investigation
- are able to ponder about the applicability range of mathematical tools

Competencies:

- are able to navigate and to develop under guidance in work situations that are not structured beforehand
- can participate in collaboration with peers in the treatment of problems of a mathematical nature
- can communicate mathematical problems and strategies leading to solutions to peers inside and outside of mathematics

Exam format: Group exam based on a written report.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.1.2 Courses MAT7A

#### **Introduction to Partial Differential Equations / Introduktion til partielle differentialligninger**

Prerequisites: The module builds on knowledge obtained by the modules Linear Algebra with Applications, Analysis 1, Analysis 2, Probability Theory from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- have knowledge about first and second order linear partial differential equations and their classification
- have knowledge about well posed and non-well posed problems
- have knowledge about solution methods for simple linear differential equations
- have knowledge about boundary value problems and initial value problems
- have knowledge about representation of solutions and regularity of solutions
- have knowledge about maximum principles and their applications
- have knowledge about elementary stochastic partial differential equations

Skills:

- are able to solve simple boundary value problems and initial value problems
- are able to apply the methods and results from the module to analyse and solve partial differential equations from areas of application

Competencies:

- are able to relate critically to models based on linear partial differential equations

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Numerical Analysis / Numerisk analyse

Prerequisites: The module builds on knowledge obtained by the modules Linear Algebra with Applications, Analysis 1, Probability Theory from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- have knowledge of floating point arithmetic, including the international standards for floating point arithmetic
- have knowledge of error analysis and stability of numerical algorithms
- have knowledge of polynomial interpolation and its application to the derivation of numerical algorithms
- have knowledge of basic results in approximation theory
- have knowledge of methods for finding zeroes of functions
- have knowledge of numerical linear algebra, in particular algorithms adapted to large sparse systems of linear equations
- have knowledge of methods for numerical differentiation, including spectral methods
- have knowledge of methods for numerical integration, including Gaussian quadrature
- have knowledge of numerical solution methods for ordinary differential equations, including spectral methods
- have knowledge of some probabilistic methods in numerical analysis, including Monte-Carlo methods

Skills:

- can implement basic numerical algorithms in different computer architectures
- can choose appropriate numerical methods to solve a given class of problems

Competencies:

- can evaluate the appropriateness of a given numerical method for solving a class of problems
- are aware of the limitations of numerical methods to solve a class of problems

Exam format: Oral exam. To be able to register for the oral exam the student must have completed assignments including one programming assignment.

Evaluation criteria: Are stated in the Joint program regulations.

**Manifolds – Differential Geometry and Topology / Mangfoldigheder – differentialgeometri og -topologi**

Prerequisites: The module builds on knowledge obtained by the modules Differential Geometry, Analysis 1 and 2, Linear Algebra with Applications from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- know central notions and results about smooth manifolds, their tangent spaces, smooth maps, both in theory and for essential examples
- have acquired knowledge about differential geometric issues among several of the following topics: vector bundles and vector fields, differential forms, Riemannian manifolds, curvature notions, Lie groups, geodesics, integration and/or dynamical systems on manifolds
- have acquired knowledge about differential topological issues among several of the following topics: regular and critical points, embedding, immersion, transversality

Skills:

- are able to present proofs of central results within differential geometry and topology
- can apply notions and methods from these subjects to important examples
- can through analysis and calculations explain properties of geometric objects

Competencies:

- are able to understand and to apply results from analysis and (linear) algebra for questions originating in geometry
- can independently formulate relevant questions and acquire new insights with point of departure in interactions of analysis, linear algebra and geometry

Exam format: Individual oral or written exam.

Evaluation criteria: Are stated in the Joint program regulations.

## Optimization / Optimizing

Prerequisites: The module builds on knowledge obtained by the modules to Analysis 2 from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

The list may include, but is not limited to, the following topics:

- know fundamentals of Unconstrained Optimization for functions of several variables: Extrema and Saddle Points
- know line search methods, conjugate gradient methods, Quasi-Newton methods
- know about calculating derivatives: Finite-Difference Method, Algorithmic Differentiation
- know about Derivative-Free Optimization
- know Least-Squares Problems
- basic knowledge about the Calculus of Variations and the Euler-Lagrange equation with applications in Economics
- basic knowledge about the Maximum Principle with applications in Economics

Skills:

- are able to utilize common and known results in the solution of concrete optimization problems
- are able to formulate and solve a numerical optimization problem
- are able to choose appropriate methods and algorithms given a concrete optimization problem

Competencies:

- are able to handle problems associated with optimization (especially in connection with applications) including relevant optimization results from the course or the literature
- are able to discuss the strengths and weaknesses of numerical optimization algorithms in relation to applications in Economics, Finance, Statistics, Engineering or Science

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

**Measure Theory and Stochastic Processes / Målteori og stokastiske prosesser**

Prerequisites: The module builds on knowledge obtained by the modules Linear Algebra with Applications, Analysis 1, Analysis 2, Probability Theory from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

**Knowledge**

- know selected topics concerning general measure theory with special focus on probability theoretical. Topics as existence and uniqueness of measures, Lebesgue-integration, Expectation and condition expectation, Radon-Nikodym's theorem, and information expressed through sigma-algebras
- know about stochastic processes in discrete and continuous time
- know about Wiener processes
- know about Martingales
- know about stochastic integrals, Ito's formula and Girsanov's theorem

**Skills**

- are able to calculate fundamental characteristics for stochastic processes
- are able to conduct a change of measure for a martingale

**Competencies**

- are able to formulate mathematical results in a correct manner by means of measure-theoretical and probabilistic argumentation
- are able to apply and mediate basic mathematics and theory related to stochastic processes
- are able to gain additional knowledge regarding probability theoretical subjects related to stochastic processes and their application in Finance

Exam format: Individual oral or written exam.

Evaluation criteria: Are stated in the Joint program regulations.

## Statistics for Duration Data / Varighedsanalyse

Prerequisites: The module builds on knowledge obtained by the modules Probability Theory, Statistical Inference for Linear Models from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- understand the special features of duration data (e.g. censoring, non-normality)
- derive the likelihood function for right-censored data
- know basic characterisations of duration data distributions such as the survival and hazard function
- be able to derive basic non-parametric estimates such as the Kaplan-Meier and Nelson-Aalen estimates
- know parametric models for duration data
- understand the assumptions underlying the Cox partial likelihood
- derive the Cox partial likelihood
- know methods of model assessment for parametric models and the Cox proportional hazards model

Skills:

- be able to identify relevant type of censoring for a specific set of duration data
- be able to estimate and interpret survival functions or cumulative hazard functions for a specific set of duration data
- be able to fit duration data using parametric or semi-parametric regression models
- be able to assess the validity of a model for a specific set of duration data

Competencies:

- be able to identify an appropriate duration data methodology for investigating a specified hypothesis of interest
- be able to interpret and critically assess results of the analysis carried out using the chosen methodology
- be able to convey the results of the analysis to a non-statistician

Exam format: Oral exam or individual ongoing during the course.

Evaluation criteria: Are stated in the Joint program regulations.

**Topics in Statistical Sciences I / Emner inden for statistisk videnskab I**

Prerequisites: The module builds on knowledge obtained by the modules Probability Theory, Statistical Inference for Linear Models from the BSc in Mathematics.

Objective:

Knowledge: The students completing the module will have gained knowledge about a number of topics from the statistical sciences at an advanced level.

The list may include, but is not limited to, the following topics:

- dynamical linear models, including the Kalman filter
- population methods, specifically evolutionary computing and genetic algorithms
- meta analysis
- robust statistical methods including non-parametric models
- factor analysis
- graphical models, including hierarchical models

Skills:

- can apply the relevant methodologies to one or more datasets by using appropriate software implementations, and interpret the output and modify the model parameters accordingly
- are able to state the underlying assumptions and argue about limitations and extendibility of the methodology in one or more specific settings
- can assess goodness-of-fit for the models where appropriate

Competencies:

- can acquire supplementary knowledge about the relevant methodologies
- can combine appropriate topics from the course to analyse a specific dataset.
- can in writing describe the methodologies, results and outcome from an analysis of a specific dataset

Exam format: Individual oral or written exam, or individual ongoing evaluation during the course.

Evaluation criteria: Are stated in the Joint program regulations.

**Topics in Statistical Science II / Emner inden for statistisk videnskab II**

Prerequisites: The module builds on knowledge obtained by the module Topics in Statistical Science I.

Objective:

Knowledge: The students completing the module will have gained knowledge about a number of topics from the statistical sciences at an advanced level. The list may include, but is not limited to, the following topics:

- state space models and hidden Markov models
- expectation-maximisation (EM) algorithm and missing data
- multivariate Gaussian distribution (and related distributions, e.g. Hotelling's  $T^2$  and Wishart distributions)
- INLA
- generalised estimating equations
- bootstrap, cross-validation and other resampling techniques

Skills:

- can apply the relevant methodologies to one or more datasets by using appropriate software implementations, and interpret the output and modify the model parameters accordingly
- are able state the underlying assumptions and argue about limitations and extendibility of the methodology in one or more specific settings
- can assess goodness-of-fit for the models where appropriate

Competencies:

- can acquire supplementary knowledge about the relevant methodologies
- can combine appropriate topics from the course to analyse a specific dataset
- can in writing describe the methodologies, results and outcome from an analysis of a specific dataset

Exam format: Individual oral or written exam, or individual ongoing evaluation during the course.

Evaluation criteria: Are stated in the Joint program regulations.

## Information and Coding Theory / Information og kodningsteori

Prerequisites: The module builds on knowledge obtained by the modules Probability Theory, Linear Algebra or similar.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- knowledge of information theoretical concepts such as entropy, mutual information, divergence, the chain rule for entropy, empirical entropy
- knowledge of lossless data compression, entropy coding, lossy data compression (rate distortion theory)
- knowledge of channel capacity and error-correcting codes
- knowledge of joint source-channel coding and the separation principle

Skills:

- are able to give a theoretical description of the entropy of a signal and in practice estimate the entropy of simple signals
- are able to design efficient entropy codes for simple signals
- are able to use information inequalities to provide bounds on optimal performance of simple systems
- are able to construct error-correcting codes with good properties and parameters
- are able to decode error-correcting codes efficiently (e.g. Reed-Solomon codes)
- understand the interaction between bitrate and distortion (reconstruction error) in connection with source coding
- understand the interaction between bitrate and error probability in connection with channel coding
- are able to perform calculations in finite fields

Competencies:

- have a good intuition and understanding of the concept of entropy and its significance regarding the information within a signal
- be able to use mathematical tools to discover and investigate the fundamental mathematical tools that describes data transmission, data reduction and data storage

Exam format: Individual oral or written exam. In order to participate in the course evaluation, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint programme regulations.

### 3.1.2 2'nd semester, MAT8A. Applied Mathematics

#### 3.1.2.1 Project MAT8A

##### **Project: Intermediate Application Oriented Mathematics / Anvendelsesorienteret matematik på mellemtrin**

Prerequisites: The module builds on knowledge obtained by the modules on the semester MAT7A.

Objective: Students who have completed the module meet the following criteria within at least one central mathematical area:

Knowledge:

- are able to explain important notions, results and theories connected to a central mathematical subject area
- can relate such results to application(s) outside the subject area

Skills:

- are able to apply a number of methods and tools from a specific mathematical subject area
- can assess whether a specific result is valid and/or whether a specific method can be applied under prescribed circumstances
- are able to form a mathematical model describing a problem amenable to a mathematical investigation and to perform a mathematical analysis that sheds light on the initiating problem
- can independently select adequate mathematical methods and tools that help treating certain questions amenable to a mathematical investigation
- are able to ponder about the applicability range of mathematical tools

Competencies:

- are able to navigate and to develop independently in work situations that are not structured beforehand
- can participate in collaboration with peers in the treatment of problems of a mathematical nature
- can communicate mathematical problems and strategies leading to solutions to peers inside and outside of mathematics

Exam format: Group exam based on a written report.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.2.2 Courses MAT8A

#### **Bayesian Inference and Mixed Models / Bayesiansk inferens og modeller med tilfældige effekter**

Prerequisites: The module builds on knowledge obtained by the module Statistical inference for linear models from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- have knowledge of the general linear model with random effects
- have knowledge of maximum likelihood inference for the general linear model with random effects
- have knowledge of prediction of random effects
- have knowledge of Bayesian inference
- have knowledge of prior distributions in Bayesian inference
- have knowledge of computational aspects of Bayesian inference

Skills:

- can for a specific dataset identify possible sources of random variation and formulate a relevant model with random effects
- can perform maximum likelihood- and Bayesian inference for the formulated model

Competencies:

- can account for methodology and practical inference for different approaches to models with random effects

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Time Series and Econometrics / Tidsrækkeanalyse og økonometri

Prerequisites: The module builds on knowledge obtained by the module Statistical Inference for Linear Models from the BSc in Mathematics.

Objectives: Students who have completed the module meet the following criteria:

Knowledge:

- know about conditioning in the multivariate normal distribution as well as ordinary and generalized least squares methods
- are able to understand a time series as a stochastic process and understand the connection between stochastic processes and dynamical systems, and in particular the Box-Jenkins models (ARMA-type models)
- know about various stationarity and non-stationarity concepts for Time Series: Weak and strong stationarity, causality, autocovariance- and autocorrelation functions, integrated models, long memory models, volatility models, and basic state-space models
- know about various modern time series and econometric models within financial econometrics and financial engineering in discrete time

Skills:

- are able to interpret the statistical and possibly econometric properties of time series
- are able to implement all phases in a classical time series analysis: Identification, estimation, diagnostic checking, prediction, and statistical/econometric interpretation
- are able to use correlograms and other graphical tools in the identification phase
- are able to apply and make themselves acquainted with new statistical methods to analyse time series

Competencies:

- are able to apply the concepts from time series in an econometric or other broader context
- are able to perform qualified econometric analyses of financial and other data including estimation and prediction using available software
- are able to reflect on the discipline's approach to academic problems at a high level and the discipline's relationship to other subject areas
- are able to involve the knowledge area in solving complex problems and thus achieve a new understanding of a given subject area

Exam form: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Spatial Statistics and Markov Chain Monte Carlo Methods / Rumlig statistik og Markovkæde Monte Carlo-metoder

Prerequisites: The module builds on knowledge obtained by the module Statistical inference for linear models from the BSc in Mathematics.

The course deals with Markov chain Monte Carlo methods as well as one or more of the three main topics within spatial statistics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- know the fundamental models and methods within the chosen main topics (geostatistics, lattice processes or spatial point processes) as well as Markov chain Monte Carlo.
- have knowledge about the following subjects within the chosen main topic(s)
  - Geostatistics:
    - Theory for second order stationary processes, variograms/covariograms, prediction and kriging, as well as model based geostatistics
  - Lattice processes:
    - Markov fields, Brook's factorisation and Hammersley-Clifford's theorem and likelihood based statistical analysis
  - Spatial point processes:
    - Poisson processes, Cox processes and Markov point processes, as well as statistical analyses based on non-parametric methods (summary statistics) and likelihood based methods
  - Markov chain Monte Carlo:
    - Fundamental theory of Markov chains with a view to simulation, Markov chain Monte Carlo methods for simulation of distributions, including the Metropolis-Hastings algorithm and the Gibbs sampler

Skills:

- are able to explain the main theoretical results from the course
- are able to perform statistical analyses of concrete datasets
- are able to simulate the examined models

Competencies:

- are able to interpret a spatial statistical model in relation to a concrete dataset and give an account of the limitations of the model with respect to describing the variation in the dataset using the theoretical results within spatial statistics
- are able to simulate distributions using Markov chain Monte Carlo methods and evaluate the output of the Markov chain

Exam format: Individual oral or written exam, or ongoing evaluation. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Graph Theory / Grafteori

Prerequisites: The module builds on knowledge obtained by the modules Discrete Mathematics and Linear Algebra with Applications from the BSc in Mathematics.

Objectives: Students who have completed the module meet the following criteria:

Knowledge:

- know about connectivity in graphs and Menger's Theorem.
- know about planarity and minors
- know about graph colouring
- know about shortest and longest cycles in graphs
- know about results in extremal graph theory
- know about probabilistic and/or (linear) algebraic methods applied to graphs

Skills:

- are able to demonstrate knowledge of survey central concepts and results from graph theory
- are able to prove central results from the module
- are able to apply relevant concepts to examples

Competencies:

- are able to independently prove small results using combinatorial reasoning possibly in conjunction with algebraic/probabilistic arguments

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Coding Theory / Kodningsteori

Prerequisites: The module builds on knowledge obtained by the modules Algebra 2 and Computer Algebra from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- know linear block codes over finite fields
- know bounds for their parameters
- know classic families of error-correcting codes, such as Reed-Solomon codes, cyclic codes, BCH codes, Hamming codes etc.
- know decoding algorithms for important families of error-correcting codes

Skills:

- are able to prove central results from the theory of error-correcting codes
- are able to apply decoding algorithms
- are able to estimate parameters of classic codes

Competencies:

- are able to apply central results from linear algebra and abstract algebra in the investigation of codes and their parameters
- are able to communicate the acquired knowledge and skills to a concrete audience
- can reason at a concrete as well as at an abstract level concerning discrete algebraic structures

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Applied Harmonic Analysis / Anvendt harmonisk analyse

Prerequisites: The module builds on knowledge obtained by the modules Analysis 1 and Linear Algebra with Applications.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- have knowledge of orthogonal functions
- have knowledge of Fourier series and their convergence, including Gibbs phenomenon
- have knowledge of Fourier integrals and convolutions, including short-time Fourier transformation and spectrograms
- have knowledge of discrete signals and analysis of such using. Harmonic analysis
- have knowledge of filter theory
- have knowledge of numerical methods in harmonic analysis
- have knowledge of Shannon's sampling theorem
- have knowledge about the use of harmonic analysis in the technical sciences

Skills:

- can calculate Fourier series for specific simple functions
- can perform a filtering of a specific signal and interpret spectrograms
- can apply harmonic analysis to well-defined problems

Qualifications:

- can evaluate the appropriateness of given harmonic analysis methods for solving a class of problems
- can acquire additional knowledge and skills in the course subject matter
- be able to reflect on the discipline's approach to academic problems at a high level and the discipline's relationship to other subject areas
- be able to involve the knowledge area in solving complex problems and thus achieve a new understanding of a given subject area

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: As stated in the Joint program regulations.

## Operators on Hilbert Spaces / Operatorer på Hilbertrum

Prerequisites: The module builds on knowledge obtained by the modules Analysis 1, Analysis 2, and Linear Algebra with Applications from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- are familiar with introductory functional analysis including completions, Banach spaces and Hilbert spaces
- have acquired an understanding of orthonormal bases
- have familiarity and understanding of bounded linear operators and their adjoints
- are familiar with the closed graph and the open mapping theorems
- are familiar with the spectral theory for bounded operators
- know the spectral theorem for self-adjoint and compact operators

Skills:

- are able to carry out proofs for central results within the theory of Banach and of Hilbert spaces
- can apply theoretical results from the module to the analysis of examples

Competencies:

- are able to apply central results from mathematical analysis and from linear algebra in the investigation of linear operators on Hilbert space and their properties
- are able independently to invoke results from functional analysis to the treatment of questions within related areas of mathematical analysis

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## **Algebraic Topology / Algebraisk topologi**

Prerequisites: The module builds on knowledge obtained by the modules Differential Geometry, Algebra 1, and Algebra 2 from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- know central notions and results from algebraic topology (notably concerning homotopy and homology)
- know important topological invariants of spaces and maps (among those fundamental groups, homology groups and induced homomorphism) and their invariance under homotopies
- have acquired insight into systematic functorial methods translating from geometric areas into combinatorial and algebraic areas

Skills:

- are able to apply and to explain notions and methods for simple examples, notably by calculation of relevant invariants
- are able to reason in correct scientific terminology and symbolic language concerning topics within algebraic topology

Competencies:

- can apply algebraic notions, methods and results to dealing with problems originating in geometry
- can independently formulate relevant questions and acquire new insights with point of departure in interactions of algebra and geometry

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## **Quantitative Finance and Computational Statistics / Quantitative Finance and Computational Statistics**

Prerequisites: The module builds on knowledge obtained by the module Statistical Inference for Linear Models and at least one programming language.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- know about quantitative software development with a focus on computational finance
- know about core models & products: stochastic volatility models, vanilla & exotic derivatives
- know about numerical treatment of stochastic differential equations (SDEs) and partial differential equations (PDEs)
- know about Monte Carlo foundations and applications
- know about Fourier transform pricing
- calibration (applied numerical optimization, market data)

Skills:

- are able to analyse a given model and apply it on market data
- are able to develop quantitative software in line with the existing practices in the financial industry
- are able to perform all stages of the verification and validation (V&V) process in quantitative software development – assessing the results obtained from a financial model

Competencies:

- are able to independently develop, analyse, and apply quantitative finance models relevant to a financial problem at hand
- are able to communicate the results of applying the models appropriate to a given financial problem to non-specialists in the financial industry
- discuss relative strengths and weaknesses of numerical methods (SDEs, PDEs, Fourier Transform) in relation to financial products (derivatives) and tasks (pricing, hedging, calibration)

Exam format: Individual oral or written exam, or individual ongoing during the course. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Financial Engineering / Financial Engineering

Prerequisites: The module builds on knowledge obtained by the module Statistical Inference in Linear Models from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

### Knowledge:

- know about financial options and derivatives, in particular exotic options and the pricing of these using numerical methods and analytical solution methods in so far these exist
- know about the fundamental principles in pricing derivatives including the underlying theoretical models
- know about standard numerical methods for pricing derivatives such as finite difference methods, Monte Carlo methods and binomial tree models
- know about the proof of the Black-Scholes-Merton plain vanilla option pricing formula
- extended knowledge about Itô calculus including quadratic variation of stochastic processes

### Skills:

- are able to calculate the price of various standard options and exotic options and derivatives
- are able to master the main steps in the proof of the Black-Scholes-Merton option pricing formula
- are able to apply Itô's lemma to various problems
- are able to use programming skills and implement numerical methods in standard software in the pricing of derivatives

### Competencies:

- are able to use standard techniques and methods to calculate prices of derivatives in practice. Moreover, the programming skills are improved
- are able to reflect on the discipline's approach to academic problems at a high level and the discipline's relationship to other subject areas
- are able to involve the knowledge area in solving complex problems and thus achieve a new understanding of a given subject area

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Data Mining / Data Mining

Prerequisites: The module builds on knowledge obtained by the module Statistical Inference for Linear Models from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- understand computer intensive techniques for validating models (cross validation and bootstrap) and can account for the variance-bias trade-off
- know of various methods for visualising high-dimensional data
- know the difference between classification and regression, and understand classification methods relying on classifications trees, prototype methods and Bayes classifiers
- know of various supervised and unsupervised methods within statistical learning
- know of association rule methods for the analysis of transaction data
- can perform link mining for network data e.g. internet pages
- have knowledge of methods to do hierarchical and partitioning cluster analysis
- know of model averaging, bagging and boosting

Skills:

- are able to identify and apply a relevant data mining algorithm in a specific context
- can identify and discuss weaknesses and strengths of different data mining algorithm in relation to a specific analysis task
- can interpret and communicate the results of a given data mining analysis to non-specialists

Competences:

- have the ability to survey potentials and limitations of different data mining software packages
- have the understanding to choose and apply specific software meeting user demands

Exam format: Individual oral or written exam, or individual ongoing during the course. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.3 3'rd semester MAT9A, Applied Mathematics

#### 3.1.3.1 Project MAT9A

##### **Project: Advanced Application Oriented Mathematics / Avanceret anvendelsesorienteret matematik**

Prerequisites: The module builds on knowledge obtained by the modules on the semester MAT8A.

Objective: Students who have completed the module meet the following criteria within at least one central mathematical area:

Knowledge:

- are able to explain important research-oriented notions, results and theories connected to a central mathematical subject area
- can relate such results to application(s) outside the subject area

Skills:

- are able to apply a number of research-oriented methods and tools from a specific mathematical subject area
- can assess whether a specific result is valid and/or whether a specific method can be applied under prescribed circumstances
- are able to form a mathematical model describing a problem amenable to a mathematical investigation and to perform a mathematical analysis that sheds light on the initiating problem
- can independently select adequate mathematical methods and tools that help treating certain questions amenable to a mathematical investigation
- are able to ponder about the applicability range of mathematical tools

Competencies:

- are able to navigate and to develop independently in work situations that are not structured beforehand
- can participate in collaboration with peers, both inside and outside of mathematics, in the treatment of problems of a mathematical nature
- can communicate mathematical problems and strategies leading to solutions to peers inside and outside of mathematics

Exam format: Group exam based on a written report.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.3.2 Courses MAT9A

These are the elective courses on MAT9A:

- Introduction to Partial Differential Equations
- Numerical Analysis
- Manifolds – Differential Geometry and Topology
- Optimization
- Measure Theory and Stochastic Processes
- Statistics for Duration Data
- Topics in Statistical Science I
- Topics in Statistical Science II
- Topics in Algebraic Geometry and Commutative Algebra
- Topics in Applied Mathematical Analysis and Geometry

Only a limited number of elective courses from the list above will be offered at each semester.

Students can only participate once in a course with a given title. In particular, they cannot follow a course if they have previously participated in a course with the same title as part of a bachelor program.

Description of the two courses “Topics in Algebraic Geometry and Commutative Algebra” and “Topics in Applied Mathematical Analysis and Geometry” can be found below. Description of the remaining courses can be found in section 3.1.1.2 Courses MAT7A.

**Topics in Algebraic Geometry and Commutative Algebra / Emner inden for algebraisk geometri og kommutiv algebra**

Prerequisites: The module builds on knowledge obtained by the modules Algebra 2 and Computer Algebra from the BSc in Mathematics.

Objectives: Students who have completed the module meet the following criteria:

Knowledge: Have knowledge about selected topics within one or more of the following areas:

- affine and projective algebraic geometry
- algebraic function field theory
- Gröbner basis theory over rings and modules
- ideals and varieties
- combinatorial commutative algebra

Skills:

- are able to prove central results from the theory of algebraic geometry and commutative algebra
- are able to employ advanced and abstract algebraic and algebraic geometric concepts and constructions
- can conduct advanced calculations within one or more of the above mentioned areas

Competencies:

- are able to apply central results from abstract algebra in the investigation of algebraic and algebraic geometric structures
- are able to communicate the acquired knowledge and skills to a concrete audience
- can reason at a concrete as well as at an abstract level concerning algebraic and algebraic geometric structures

Exam format: Individual oral or written exam or individual ongoing during the course.

Evaluation criteria: Are stated in the Joint program regulations.

**Topics in Applied Mathematical Analysis and Geometry/Emner inden for anvendt matematisk analyse og geometri**

Prerequisites: The module builds on knowledge obtained by the modules Analysis I and II, Differential Geometry and Integration Theory from the BSc in Mathematics.

Objectives: Students who have completed the module meet the following criteria:

Knowledge: Have gained experience within one or several of the following areas:

- calculus of variations
- harmonic analysis
- Spectral theory
- mathematical physics
- (directed) algebraic topology
- Riemannian geometry

Skills:

- are able to present proofs of central results within one or several of the above mentioned areas
- can apply notions and methods from the area(s) to important examples

Competencies:

- are able to describe and to analyse models from an area of application using key mathematical tools from the area(s)
- can reason at a concrete as well as at an abstract level concerning structures from analysis, geometry and/or topology

Exam format: Individual oral or written exam, or individual continuous evaluation during the course.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.4 4<sup>th</sup> semester MAT10A, Applied Mathematics

#### **Project: Master's Thesis / Kandidatspeciale**

The student has the possibility to write a Long Master's Thesis (over 2 semesters: 60 ECTS), if the thesis is of experimental character. The amount of experimental work must reflect the allotted ECTS.

Objectives: Students who have completed the module meet the following criteria within at least one central mathematical/statistical area:

#### Knowledge:

- have expert understanding within one or a few selected elements of a central mathematical subject area based on high level research, or has a broader insight into a central mathematical subject area regarding theories and methods as well as central elements and their interrelationships
- must be able to understand and on a scientific basis reflect upon the knowledge of the mathematical subject area and be able to identify scientific problems

#### Skills:

- must be able to identify, formulate and analyse a scientific problem independently, systematically and critically
- must be able to relate the problem to the mathematical subject area, including explaining the choices that have been made in connection to the delimitation of the problem
- must be able to independently make and justify the choice of mathematical theories and methods
- must be able to independently and critically evaluate the chosen theories and methods as well as the analyses, results and conclusions in the project, both during and at the end of the project period
- must be able to evaluate and choose between the scientific theories, methods, tools, and general skills within the mathematical subject area

#### Competencies:

- must be able to control work and development situations which are complex, unpredictable and require new mathematical models or methods for solution
- must be able to initiate and complete mathematically oriented collaborations, and if relevant also interdisciplinary collaborations, as well as assume professional responsibility
- must be able to independently assume responsibility for own professional development and specialisation

Exam format: Group exam based on project report.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.5 MAT7M Mathematics as a central subject (Matematik som centralt fag)

#### 3.1.5.1 Project MAT7M

##### **Project: Statistical Modelling and Analysis / Statistisk modellering og analyse**

Prerequisites: The module builds on knowledge obtained by the module Probability Theory from the BSc in Mathematics, and the course module Statistical Inference for Linear Models, which must be followed in parallel.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- have knowledge about formulating a statistical model initialised by a concrete problem from a subject area outside mathematics
- have knowledge about how to perform statistical inference for a generalised linear model
- have knowledge about how to perform model checking

Skills:

- are able to formulate a relevant generalised linear model initialised by a concrete problem while taking account of the available data
- are able to apply statistical software for implementing and analysing a concrete statistical model
- are able to evaluate the validity of obtained results

Competencies:

- are able to communicate the results of a statistical analysis to non-statisticians with interest in the treated problem
- are able to convey obtained knowledge and skills to a predetermined audience
- are able to reason about the origin and application of mathematical concepts and tools in a given social, historical or technological context (scientific theoretical dimension)
- are able to independently develop generalised linear models fitting data
- know scientific theoretical aspects relating to generalisability of statistical analyses
- are able to reflect on the discipline's approach to academic problems at a high level and the discipline's relationship to other subject areas
- are able to involve the knowledge area in solving complex problems and thus achieve a new understanding of a given subject area

Exam format: Group exam based on project report.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.5.2 Courses MAT7M

#### Statistical Inference for Linear Models / Statistisk inferens for lineære modeller

Prerequisites: The module builds on knowledge obtained by the module Probability Theory from the BSc in Mathematics.

Objective: Students completing the module will fulfil the following

Knowledge:

- have knowledge of the steps in a statistical analysis
- know of the exponential family of distributions
- have knowledge of generalised linear models, in particular linear normal models
- have knowledge of estimation, including maximum likelihood estimation
- have knowledge of statistical inference, including hypothesis test
- know of examples for model assessment
- know of and can apply relevant statistical software

Skills:

- can by the use of relevant statistical software, carry out a statistical analysis of a dataset med with departure in a given generalised linear model, including estimation, model assessment, hypothesis test and interpretation
- can account for the mathematical properties for a given generalised linear model

Competencies:

- can acquire supplementary knowledge and skills within the course's topics
- can formulate correct statements in statistical and probability theoretical terms
- have knowledge about arguments from philosophy of science that underlies the formulation and tests of scientific hypotheses within statistical inference

Exam format: Individual oral or written exam. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Computer Algebra / Computeralgebra

Prerequisites: The module builds on knowledge obtained by the modules Linear Algebra with Applications, Algebra 1 and Algebra 2 from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- know algorithms for fast multiplication of integers and polynomials, including the FFT
- are able to describe and analyse the EEA (extended Euclidean algorithm) to calculate and describe the greatest common divisor
- know about modular arithmetic and several of its applications
- know methods for factorizing two integers and/or polynomials and their/its applications
- know essential data structures for polynomials and finite fields
- have some knowledge on advance topics like Gröbner bases and their applications, symbolic integration, or symbolic summation

Skills:

- are able to use graphical tools of a computer algebra system
- are able to implement simple algorithms and calculations in a computer algebra system
- can simplify and transform mathematical structures using a computer algebra system
- are able to analyse the computational complexity of algorithms

Competencies:

- in simple cases, are able to decide whether a computer algebra system can be used to solve a specific mathematical problem
- are able to implement and interpret simple algorithms for solving mathematical problems
- are able to communicate the acquired knowledge and skills to a concrete audience
- are able to judge the use of computer algebra systems for disseminating mathematics

Exam format: Individual oral or written exam, or individual ongoing during the course. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

## Differential Geometry / Differentialgeometri

Prerequisites: The module builds on knowledge obtained by the modules Analysis and Linear Algebra with Applications from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- are able to characterize smooth curves by curvature and torsion
- are able to describe a regular surface and its tangent planes
- know about smooth maps and their differentials
- know the two fundamental forms and their application to geometric analysis
- are able to describe and interpret important curvature notions on surfaces and relate those to each other
- know about geodesic curves, their characterization and properties
- can give examples of global geometric invariants for regular surfaces.

Skills:

- are able to prove central results from the theory of curves and surfaces
- are able to calculate important characteristic numerical invariants for curves and surfaces
- are able to apply theoretical results from the syllabus to the analysis of concrete examples

Competencies:

- are able to apply central results from analysis and from linear algebra in the investigation of geometric properties and invariants
- are able to decide whether certain geometric constructions are possible or not, using invariants
- are able to comment on the interaction between methods from various mathematical areas, in particular from analysis and from linear algebra, in the analysis of theoretic and practical problems of a geometric nature (theory of science dimension)

Exam format: Individual oral or written exam, or individual ongoing during the course. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.6 MAT8M Mathematics as a central subject (Matematik som centralt fag)

#### 3.1.6.1 Project MAT8M

##### **Project: Mathematics with Applications / Matematik med anvendelser**

Prerequisites: The module builds on knowledge obtained by the modules on the semester MAT7M.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- have research based knowledge about theory, methods and practice within one or more mathematical subjects
- must be able to understand and reflect upon theory, scientific methods and practice

Skills:

- are able to apply the methods and tools from the subject area(s)
- are able to evaluate the theoretical and practical problems within the subject area(s) as well as justify and choose the relevant models for analysis and solutions
- are able to convey technical problems and models for solution to both colleagues and non-specialists

Competencies:

- are able to handle complex and development oriented situations in study or work contexts
- are able to independently enter technical and interdisciplinary with a professional approach
- are able to identify own learning needs and structure own learning in different learning environments
- are able to reflect on the discipline's approach to academic problems at a high level and the discipline's relationship to other subject areas
- are able to involve the knowledge area in solving complex problems and thus achieve a new understanding of a given subject area

Exam format: Oral evaluation based on project report.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.6.2 Courses MAT8M

The students select courses equivalent to 10 ECTS. The students must follow the course “Integration Theory”, see description below. For description of the other courses on this semester, please see section 3.1.2.2. (MAT8A).

- Bayesian Inference and Mixed Models
- Time Series and Econometrics
- Spatial Statistics and Markov Chain Monte Carlo Methods
- Graph Theory
- Applied Harmonic Analysis
- Coding Theory
- Operators on Hilbert Spaces
- Algebraic Topology
- Quantitative Finance and Computational Statistics
- Financial Engineering
- Data Mining

Only a limited number of elective courses from the list above will be offered at each semester. Students can only participate once in a course with a given title. In particular, they cannot follow a course if they have previously participated in a course with the same title as part of a bachelor program.

## Integration Theory / Integrationsteori

Prerequisites: The module builds on knowledge obtained by the module Analysis 1 from the BSc in Mathematics.

Objective: Students who have completed the module meet the following criteria:

Knowledge:

- are familiar with abstract measures and with sigma algebras, counting measures and probability measures
- are familiar with measurable maps and Borel functions
- are familiar with the Lebesgue integral and with monotone and dominated convergence
- are familiar with properties of the Lebesgue measure and its construction
- are familiar with the construction of product measures and with Tonelli's and Fubini's theorems
- are familiar with the completeness of Lebesgue spaces and with Hölder and Minkowski inequalities
- are familiar with convolutions, Fourier transformation and Plancherel's isometry

Skills:

- are able to prove central results from the theory of Lebesgue integration
- are able to apply the theoretical results contained in the module to concrete examples

Competencies:

- are able to argue correctly for measurability and integrability for both general and concrete examples
- can invoke relevant measure spaces and corresponding results to treat questions concerning integrals

Exam format: Individual oral or written exam, or individual ongoing during the course. In order to participate in the exam, students must have actively participated in course progress by way of one or several independent oral and/or written contributions.

Evaluation criteria: Are stated in the Joint program regulations.

### 3.1.7 MAT9M Mathematics as a central subject (Matematik som centralt fag)

The students follow the second subject.

### 3.1.8 MAT10M Mathematics as a central subject (Matematik som centralt fag)

Cf. section 3.1.4.

### 3.1.9 Mathematics as a central/elective subject (Matematik som sidefag)

#### 3.1.9.1 Project - Mathematics as elective subject (Matematik som sidefag)

##### 3<sup>rd</sup> semester, MAT9M (Same project as MAT7M)

Cf. section 3.1.5.1.

#### 3.1.9.2 Courses. Mathematics as elective subject (Matematik som sidefag)

Description of the courses on this semester:

- Statistical Inference for Linear Models
- Computer algebra
- Differential Geometry

can be found in section 3.1.5.2 (MAT7M).

## ***Chapter 4: Entry into Force, Interim Provisions and Revision***

The curriculum is approved by the Dean of The Faculty of Engineering and Science and enters into force as of 1<sup>st</sup> September 2017.

Students who wish to complete their studies under the previous curriculum from 2016 must conclude their education by the summer examination period 2018 at the latest, since examinations under the previous curriculum are not offered after this time.

## ***Chapter 5: Other Provisions***

### **5.1 Rules concerning written work, including the Master's Thesis**

In the assessment of all written work, regardless of the language it is written in, weight is also given to the student's spelling and formulation ability, in addition to the academic content. Orthographic and grammatical correctness as well as stylistic proficiency are taken as a basis for the evaluation of language performance. Language performance must always be included as an independent dimension of the total evaluation. However, no examination can be assessed as 'Pass' on the basis of good language performance alone; similarly, an examination normally cannot be assessed as 'Fail' on the basis of poor language performance alone.

The Board of Studies can grant exemption from this in special cases (e.g., dyslexia or a native language other than Danish).

The Master's thesis must include an English summary.<sup>2</sup> If the project is written in English, the summary must be in Danish.<sup>3</sup> The summary must be at least 1 page and not more than 2 pages. The summary is included in the evaluation of the project as a whole.

## **5.2 Rules concerning credit transfer (merit), including the possibility for choice of modules that are part of another program at a university in Denmark or abroad**

The Board of Studies can approve successfully completed (passed) program elements from other Master's programs in lieu of program elements in this program (credit transfer). The Board of Studies can also approve successfully completed (passed) program elements from another Danish program or a program outside of Denmark at the same level in lieu of program elements within this curriculum. Decisions on credit transfer are made by the Board of Studies based on an academic assessment. See the Joint program regulations for the rules on credit transfer.

## **5.3 Rules for examinations**

The rules for examinations are stated in the Examination Policies and Procedures published by The Technical Faculty of IT and Design, The Faculty of Engineering and Science, and The Faculty of Medicine on their website.

## **5.4 Exemption**

In exceptional circumstances, the Board of Studies study can grant exemption from those parts of the curriculum that are not stipulated by law or ministerial order. Exemption regarding an examination applies to the immediate examination.

## **5.5 Rules and requirements for the reading of texts**

At programs that are taught in Danish, it is assumed that the student can read academic texts in modern Danish, Norwegian, Swedish and English and use reference works, etc., in other European languages. At programs taught in English, it is assumed that the student can read academic text and use reference works, etc., in English.

## **5.6 Additional information**

The current version of the curriculum is published on the Board of Studies' website, including more detailed information about the program, including exams.

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<sup>2</sup> Or another foreign language (upon approval from the Board of Studies).

<sup>3</sup> The Board of Studies can grant exemption from this.